



Mark Scheme (Results)

October 2020

Pearson Edexcel International A Level
In Further Pure Mathematics F3
(WFM03/01)

Question Number	Scheme	Notes	Marks
1(a)	$4 \sinh^3 x + 3 \sinh x = 4 \left(\frac{e^x - e^{-x}}{2} \right)^3 + 3 \left(\frac{e^x - e^{-x}}{2} \right)$ $= 4 \left(\frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8} \right) + 3 \left(\frac{e^x - e^{-x}}{2} \right)$ Uses $\sinh x = \frac{e^x - e^{-x}}{2}$ on both $\sinh$ terms and attempts to cube the bracket (min accepted is a linear x a quadratic bracket)		M1
	$= \frac{1}{2} e^{3x} - \frac{3}{2} e^x + \frac{3}{2} e^{-x} - \frac{1}{2} e^{-3x} + \frac{3}{2} e^x - \frac{3}{2} e^{-x}$ $= \frac{e^{3x} - e^{-3x}}{2} = \sinh 3x^*$		A1*
			(2)
(b)	$\sinh 3x = 19 \sinh x \Rightarrow 4 \sinh^3 x + 3 \sinh x = 19 \sinh x$ $\Rightarrow 4 \sinh^3 x - 16 \sinh x = 0$ Uses the result from (a) and combines terms		M1
	$(\sinh x = 0 \text{ or } \sinh^2 x = 4)$	$\sinh^2 x = 4 \text{ or } \sinh x = (\pm)2$	A1
	$(0, 0)$	States the origin as one intersection	B1
	$\ln(2 + \sqrt{5})$ and $-\ln(2 + \sqrt{5})$	Two correct non-zero x values (allow e.g. $\ln(-2 + \sqrt{5})$ for $-\ln(2 + \sqrt{5})$)	A1
	$(\ln(2 + \sqrt{5}), 38)$ and $(-\ln(2 + \sqrt{5}), -38)$	Two correct points (allow e.g. $\ln(-2 + \sqrt{5})$ for $-\ln(2 + \sqrt{5})$)	A1
			(5)
Alternative for (b) using exponentials			
	$\sinh 3x = 19 \sinh x \Rightarrow \frac{e^{3x} - e^{-3x}}{2} = \frac{19(e^x - e^{-x})}{2} \Rightarrow \dots$ Substitutes the correct exponential forms and collects terms to one side		M1
	$\Rightarrow e^{6x} - 19e^{4x} + 19e^{2x} - 1 = 0$	Correct equation (or equivalent)	A1
	$(0, 0)$	States the origin as one intersection	B1
	$\frac{1}{2} \ln(9 + 4\sqrt{5})$ or $\frac{1}{2} \ln(9 - 4\sqrt{5})$	Two correct non-zero x values (oe)	A1
	$\left(\frac{1}{2} \ln(9 + 4\sqrt{5}), 38\right)$ and $\left(\frac{1}{2} \ln(9 - 4\sqrt{5}), -38\right)$	Two correct points (oe)	A1
			Total 7

Question Number	Scheme	Notes	Marks
2(i)	$3x^2 + 12x + 24 = 3(x^2 + 4x + 8)$ $= 3((x+2)^2 + 4)$	Obtains $3((x+2)^2 + \dots)$ or $3(x+2)^2 + \dots$ Must include 3 now or later	M1
	$3((x+2)^2 + 4)$ or $3(x+2)^2 + 12$		A1
	$\int \frac{1}{3x^2 + 12x + 24} dx = \frac{1}{3} \int \frac{1}{(x+2)^2 + 4} dx = \frac{1}{6} \arctan \frac{x+2}{2} (+c)$ M1: Use of arctan A1: Fully correct expression (condone omission of + c)		M1A1
			(4)
(ii)	$27 - 6x - x^2 = -(x^2 + 6x - 27)$ $= -((x+3)^2 - 36)$	Obtains $-((x+3)^2 + \dots)$ or $-(x+3)^2 + \dots$	M1
	$-((x+3)^2 - 36)$ or $36 - (x+3)^2$		A1
	$\int \frac{1}{\sqrt{27 - 6x - x^2}} dx = \int \frac{1}{\sqrt{36 - (x+3)^2}} dx = \arcsin\left(\frac{x+3}{6}\right) (+c)$ (Or $= -\arccos\left(\frac{x+3}{6}\right) (+c)$) M1: Use of arcsin (or – arccos) A1: Fully correct expression (condone omission of + c)		M1A1
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
3	$\mathbf{M} = \begin{pmatrix} 3 & -4 & k \\ 1 & -2 & k \\ 1 & -5 & 5 \end{pmatrix}$		
(a)	$ \mathbf{M} - \lambda \mathbf{I} = \mathbf{M} = \begin{vmatrix} 3-\lambda & -4 & k \\ 1 & -2-\lambda & k \\ 1 & -5 & 5-\lambda \end{vmatrix} = \begin{vmatrix} 0 & -4 & k \\ 1 & -5 & k \\ 1 & -5 & 2 \end{vmatrix}$ $(0) + 4[2-k] + k[-5+5]$ Attempts $\mathbf{M} - \lambda \mathbf{I}$ using $\lambda = 3$		M1
	$(0) + 4[2-k] + k[-5+5] = 0 \Rightarrow k = \dots$ Uses $\mathbf{M} - \lambda \mathbf{I} = 0$ and solves for k		M1
	$k = 2$	Cao	A1
			(3)
(b)	$(3-\lambda)[(\lambda+2)(\lambda-5)+10] + 4(5-\lambda-2) + 2(-5+2+\lambda) = 0$ Attempts $\mathbf{M} - \lambda \mathbf{I} = 0$ using their value of k		M1
	$\Rightarrow (3-\lambda)[(\lambda+2)(\lambda-5)+12] = 0$ $(\lambda+2)(\lambda-5)+12 \Rightarrow \lambda^2 - 3\lambda + 2 = 0 \Rightarrow (\lambda-2)(\lambda-1) = 0 \Rightarrow \lambda = \dots$ Uses $\lambda = 3$ as a factor to obtain and solve a 3TQ to find the other eigenvalues (Alternatively may use calculator to solve $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$)		M1
	$\lambda = 1, 2$	Correct values	A1
			(3)
(c)	$\begin{pmatrix} 3 & -4 & 2 \\ 1 & -2 & 2 \\ 1 & -5 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{cases} 3x - 4y + 2z = 3x \\ x - 2y + 2z = 3y \\ x - 5y + 5z = 3z \end{cases}$	Uses the eigenvalue 3 and their k to form at least 2 equations in x , y and z	M1
	$\alpha \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \text{ } (\alpha \text{ a constant})$	Any correct eigenvector. Allow any constant multiple of $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	A1
	$\frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$	Correct normalised vector	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
4.	$I_n = \int x^n \cos x \, dx$		
(a)	$\int x^n \cos x \, dx = x^n \sin x - \int nx^{n-1} \sin x \, dx$ M1: Parts in the correct direction A1: Correct expression		M1A1
	$= x^n \sin x - \left\{ -nx^{n-1} \cos x + \int n(n-1)x^{n-2} \cos x \, dx \right\}$ Uses integration by parts again (dependent on the first M)		dM1
	$= x^n \sin x + nx^{n-1} \cos x - n(n-1)I_{n-2}^*$ Fully correct proof with no errors		A1*
			(4)
ALT			
	$I_n = \int x^n \cos x \, dx = \int x^{n-1} (x \cos x) \, dx$		
	$= x^n \sin x + x^{n-1} \cos x - (n-1) \int x^{n-2} (x \sin x + \cos x) \, dx$ M1: Parts in the correct direction A1: Correct expression		M1A1
	$= x^n \sin x + x^{n-1} \cos x - (n-1) \int x^{n-1} \sin x \, dx - (n-1)I_{n-2}$		
	$= x^n \sin x + x^{n-1} \cos x - (n-1) \left\{ -x^{n-1} \cos x + (n-1)I_{n-2} \right\} - (n-1)I_{n-2}$ Uses integration by parts again (dependent on the first M)		dM1
	$= x^n \sin x + nx^{n-1} \cos x - n(n-1)I_{n-2}^*$ Fully correct proof with no errors		A1*
(b)	$I_0 = \sin x \quad (+k)$		B1
	$I_4 = x^4 \sin x + 4x^3 \cos x - 12I_2$	Applies the reduction formula once for I_4 or I_2	M1
	$= x^4 \sin x + 4x^3 \cos x - 12(x^2 \sin x + 2x \cos x - 2I_0)$ Applies the reduction formula again and obtains an expression for I_4 which can include I_0 but not I_2		M1
	$= (x^4 - 12x^2 + 24) \sin x + (4x^3 - 24x) \cos x + c$ Award A1 for either bracket and A1 for the other If the answer is not factorised but is otherwise correct, award A1A0		A1A1
			(5)
			Total 9

Question Number	Scheme	Notes	Marks
5	$\frac{x^2}{25} - \frac{y^2}{4} = 1 \quad y = mx + c$		
(a)	$\frac{x^2}{25} - \frac{(mx+c)^2}{4} = 1 \Rightarrow 4x^2 - 25(m^2x^2 + 2cmx + c^2) = 100$ Substitutes to obtain a quadratic in x and eliminates fractions		M1
	$4x^2 - 25(m^2x^2 + 2cmx + c^2) = 100$ $(\Rightarrow (25m^2 - 4)x^2 + 50cmx + 25c^2 + 100 = 0)$ Correct 3TQ		A1
	$"b^2 = 4ac" \Rightarrow (50cm)^2 = 4(25m^2 - 4)(25c^2 + 100)$ Uses ' $b^2 = 4ac$ ' or equivalent		M1
	$2500c^2m^2 = 2500c^2m^2 + 10000m^2 - 400c^2 - 1600$ $10000m^2 = 400c^2 + 1600$ $25m^2 = c^2 + 4^*$ Fully correct proof with no errors		A1*
			(4)
ALT 1	Using hyperbolic parameters:		
	$x = 5 \cosh t, y = 2 \sinh t \Rightarrow \frac{dy}{dx} = \frac{2 \cosh t}{5 \sinh t}$		
	$\frac{2 \cosh t}{5 \sinh t}(x - 5 \cosh t) = y - 2 \sinh t$ M1: Attempts the equation of the tangent A1: Correct equation (no simplification needed)		M1A1
	$y = \frac{2 \cosh t}{5 \sinh t}x - \frac{2 \cosh^2 t - 25 \sinh^2 t}{\sinh t}$		
	$25m^2 = \frac{4 \cosh^2 t}{\sinh^2 t}, 4 + c^2 = 4 + \frac{4}{\sinh^2 t} = \frac{4(\sinh^2 t + 1)}{\sinh^2 t} = \frac{4 \cosh^2 t}{\sinh^2 t}$ Extracts $25m^2$ and $4 + c^2$ from their equation		M1
	$\therefore 25m^2 = 4 + c^2^*$ Fully correct proof with no errors		A1*
			(4)
ALT 2	Using trigonometric parameters:		
	$x = 5 \sec t, y = 2 \tan t \Rightarrow \frac{dy}{dx} = \frac{2 \sec t}{5 \tan t}$		
	$\frac{2 \sec t}{5 \tan t}(x - 5 \sec t) = y - 2 \tan t$ M1: Attempts the equation of the tangent A1: Correct equation (no simplification needed)		M1A1
	$y = \frac{2 \sec t}{5 \tan t}x + \frac{2 \tan^2 t - 2 \sec^2 t}{\tan t}$		
	$25m^2 = \frac{4 \sec^2 t}{\tan^2 t} = \frac{4}{\sin^2 t} \quad 4 + c^2 = 4 \left(1 + \frac{1}{\tan^2 t}\right) = 4 \left(\frac{\sin^2 t + \cos^2 t}{\sin^2 t}\right) = \frac{4}{\sin^2 t}$ Extracts $25m^2$ and $4 + c^2$ from their equation		M1
	$\therefore 25m^2 = 4 + c^2^*$ Fully correct proof with no errors		A1*
			(4)

(b)	$25m^2 = c^2 + 4 \text{ and } 2 = m + c$ $25m^2 = (2 - m)^2 + 4 \text{ or } 25(2 - c)^2 = c^2 + 4$ Uses the given hyperbola and the straight line with the result from (a) to obtain an equation in m or c		M1
	$24m^2 + 4m - 8 = 0$ or $24c^2 - 100c + 96 = 0$	Correct 3TQ in m or c	A1
	$24m^2 + 4m - 8 = 0 \Rightarrow m = \frac{1}{2}, -\frac{2}{3}$ Or $24c^2 - 100c + 96 = 0 \Rightarrow c = \frac{3}{2}, \frac{8}{3}$	Solves their 3TQ in m or c	M1
	$y = \frac{1}{2}x + \frac{3}{2} \text{ or } y = -\frac{2}{3}x + \frac{8}{3}$	One correct tangent	A1
	$y = \frac{1}{2}x + \frac{3}{2} \text{ and } y = -\frac{2}{3}x + \frac{8}{3}$	Both correct tangents	A1
			(5)
(c)	$m = \frac{1}{2}, c = \frac{3}{2} \Rightarrow \frac{9}{4}x^2 + \frac{75}{2}x + \frac{625}{4} = 0 \Rightarrow x = \dots$ or $m = -\frac{2}{3}, c = \frac{8}{3} \Rightarrow \frac{64}{9}x^2 - \frac{800}{9}x + \frac{2500}{9} = 0 \Rightarrow x = \dots$ Uses one of their m and c pairs and solves for x		M1
	$x = -\frac{25}{3}, y = -\frac{8}{3} \text{ or } x = \frac{25}{4}, y = -\frac{3}{2}$	One correct point	A1
	$x = -\frac{25}{3}, y = -\frac{8}{3} \text{ and } x = \frac{25}{4}, y = -\frac{3}{2}$	Both correct points	A1
			(3)
			Total 12

Question Number	Scheme	Notes	Marks
6(a)	$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & a \end{pmatrix}$		
	$ \mathbf{A} = a - 2 + a - 1 + 2 - 1 (= 2a - 2)$	Correct determinant in any form	B1
	$\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & a \end{pmatrix} \rightarrow \begin{pmatrix} a-2 & a-1 & 1 \\ -a-2 & a-1 & 3 \\ -2 & 0 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} a-2 & 1-a & 1 \\ a+2 & a-1 & -3 \\ -2 & 0 & 2 \end{pmatrix}$ Applies the correct method to reach at least a matrix of cofactors 2 correct rows or 2 correct columns needed		M1
	$\begin{pmatrix} a-2 & 1-a & 1 \\ a+2 & a-1 & -3 \\ -2 & 0 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} a-2 & a+2 & -2 \\ 1-a & a-1 & 0 \\ 1 & -3 & 2 \end{pmatrix}$ Correct transpose of cofactors		A1
	$\mathbf{A}^{-1} = \frac{1}{2a-2} \begin{pmatrix} a-2 & a+2 & -2 \\ 1-a & a-1 & 0 \\ 1 & -3 & 2 \end{pmatrix}$	Correct inverse	A1
			(4)
(b)	$a = 4 \Rightarrow \mathbf{A}^{-1} = \frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$= \frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix} \begin{pmatrix} 12-6\lambda \\ 4+2\lambda \\ 6+3\lambda \end{pmatrix} = \dots$	Attempt to multiply the parametric form of l_2 by their inverse	M1
	$= \begin{pmatrix} 6-\lambda \\ -4+4\lambda \\ 2-\lambda \end{pmatrix}$	Correct parametric form	A1
	$\mathbf{r} = \begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$	Correct equation (allow equivalent forms) but if given as $l = \dots$ award A0	A1
			(4)
			Total 8

	Alternatives for (b)		
(i)	$a = 4 \Rightarrow \mathbf{A}^{-1} = \frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$\frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix} \begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 36 \\ -24 \\ 12 \end{pmatrix}$	Attempt $\mathbf{A}^{-1}$ (point on l_2) and $\mathbf{A}^{-1}$ (direction of l_2)	M1
	$\frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix} \begin{pmatrix} -6 \\ 2 \\ 3 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} -6 \\ 24 \\ -6 \end{pmatrix}$	Both correct (NB No ft)	A1
	$\mathbf{r} = \begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$	Correct equation (allow equivalent forms) but if given as $l = \dots$ award A0	A1
			(4)
(ii)	$a = 4 \Rightarrow \mathbf{A}^{-1} = \frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$\frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix} \begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 36 \\ -24 \\ 12 \end{pmatrix}$	Attempt $\mathbf{A}^{-1}$ (point on l_2) for 2 points	M1
	$\frac{1}{6} \begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \\ 9 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 30 \\ 0 \\ 6 \end{pmatrix}$	Both correct (NB No ft)	A1
	$\mathbf{r} = \begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$	Obtain the direction vector and deduce correct equation (allow equivalent forms) but if given as $l = \dots$ award A0	A1
			(4)

Question Number	Scheme	Notes	Marks
7	$x = \cosh t + t, \quad y = \cosh t - t$		
(a)	$\frac{dx}{dt} = \sinh t + 1, \quad \frac{dy}{dt} = \sinh t - 1$	Correct derivatives	B1
	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \sinh^2 t + 2\sinh t + 1 + \sinh^2 t - 2\sinh t + 1$ $= 2\sinh^2 t + 2$ M1: Squares correctly, cancels and collects terms		M1
	$= 2(1 + \sinh^2 t) = 2\cosh^2 t^*$	Uses $\cosh^2 t = 1 + \sinh^2 t$ to complete the proof with no errors	A1*
			(3)
(b)	$S = 2\pi \int y \, ds = 2\pi \int (\cosh t - t)\sqrt{2} \cosh t \, dt$	Uses $S = 2\pi \int y \, ds$ with the given y and the result from part (a)	M1
	$= 2\sqrt{2}\pi \int_0^{\ln 3} (\cosh^2 t - t \cosh t) \, dt^*$	Correct proof with no errors	A1*
			(2)
(c)	$\int \cosh^2 t \, dt = \int \pm \frac{1}{2} \pm \frac{1}{2} \cosh 2t \, dt$	Uses $\cosh^2 t = \pm \frac{1}{2} \pm \frac{1}{2} \cosh 2t$	M1
	$\int t \cosh t \, dt = t \sinh t - \int \sinh t \, dt$	Attempts integration by parts the right way round on $t \cosh t$	M1
		Correct expression	A1
	$S = (2\sqrt{2}\pi) \int (\cosh^2 t - t \cosh t) \, dt = (2\sqrt{2}\pi) \left[\frac{1}{2}t + \frac{1}{4}\sinh 2t - t \sinh t + \cosh t \right]$ A1: 2 correct terms A1: All correct		A1A1
	$(S =) 2\sqrt{2}\pi \left\{ \left(\frac{1}{2}\ln 3 + \frac{10}{9} - \frac{4}{3}\ln 3 + \frac{5}{3} \right) - (1) \right\}$ dM1: Correct use of limits 0 and ln3 depends on both preceding M marks		dM1
	$S = \frac{1}{9}\sqrt{2}\pi(32 - 15\ln 3)$	cao	A1 (7)
			Total 12
	Alternative for (c)		
	$\int \cosh^2 t \, dt = \int \left(\frac{e^t + e^{-t}}{2} \right)^2 \, dt$ $= \frac{1}{4} \int (e^{2t} + 2 + e^{-2t}) \, dt$	Substitutes the exponential form and attempts to square	M1
	$\int t \cosh t \, dt = \frac{1}{2} \int t(e^t + e^{-t}) \, dt$ $= \frac{1}{2}te^t - \frac{1}{2} \int te^t \, dt - \left\{ \frac{1}{2}te^{-t} - \frac{1}{2} \int e^{-t} \, dt \right\}$	Substitutes the exponential form and attempts integration by parts the right way round Correct expression	M1 A1
	$(S =) (2\sqrt{2}\pi) \left\{ \frac{1}{4} \left(\frac{1}{2}e^{2t} + 2t - \frac{1}{2}e^{-2t} \right) - \frac{1}{2}te^t + \frac{1}{2}e^t + \frac{1}{2}te^{-t} - \frac{1}{2}e^{-t} \right\}$ A1: either integral correct A1: other integral correct but both must be in a complete expression for S		A1A1
	Depends on both M marks above	Correct use of limits 0 and ln3	dM1
	$S = \frac{1}{9}\sqrt{2}\pi(32 - 15\ln 3)$	cao	A1

	Alternative for the first 3 marks of (c)		
	$= 2\sqrt{2}\pi \int (\cosh^2 t - t \cosh t) dt$ $= 2\sqrt{2}\pi \int \cosh t (\cosh t - t) dt$		
	$2\sqrt{2}\pi \left(\left[\sinh t (\cosh t - t) \right] - \int \sinh t (\sinh t - 1) dt \right)$		
	$2\sqrt{2}\pi \left(\left[\sinh t (\cosh t - t) \right] - \left[\cosh t (\sinh t - 1) \right] + \int \cosh^2 t dt \right)$ M1 (2nd on e-PEN): Use parts twice A1 Correct expression		M1A1
	$\int \cosh^2 t dt = \int \pm \frac{1}{2} \pm \frac{1}{2} \cosh 2t dt$	Uses $\cosh^2 t = \pm \frac{1}{2} \pm \frac{1}{2} \cosh 2t$	M1 (1st on e-PEN)
	Rest as main scheme		

Question Number	Scheme	Notes	Marks
8(a)	$\mathbf{n} = \begin{pmatrix} 1 \\ -5 \\ 3 \end{pmatrix} \times \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} -10+6 \\ -(2-9) \\ -2+15 \end{pmatrix}$	Attempt vector product between normal vectors	M1
	$= \begin{pmatrix} -4 \\ 7 \\ 13 \end{pmatrix}$	Correct vector	A1
	$x = 0 \Rightarrow -5y + 3z = 11, \quad -2y + 2z = 7$ $\Rightarrow y = -\frac{1}{4}, z = \frac{13}{4}$ or $y = 0 \Rightarrow x + 3z = 11, \quad 3x + 2z = 7$ $\Rightarrow x = -\frac{1}{7}, z = \frac{26}{7}$ or $z = 0 \Rightarrow x - 5y = 11, 3x - 2y = 7$ $\Rightarrow x = 1, y = -2$	Correct strategy to find a point on l	M1
		Correct position vector of point on l	A1
	$\mathbf{r} = \mathbf{i} - 2\mathbf{j} + \lambda(-4\mathbf{i} + 7\mathbf{j} + 13\mathbf{k})$	Correct equation. (follow through their position and direction vectors but must be " $\mathbf{r} =$ ")	A1ft
			(5)
ALT	$x = 11 + 5y - 3z$		
	$3x - 2y + 2z = 7 \Rightarrow 3(11 + 5y - 3z) - 2y + 2z = 7$ $\Rightarrow y - \frac{7z}{13} = -\frac{26}{13} \quad \left(z = \frac{13y + 26}{7} \right)$ Eliminate one variable		M1
	$x = 11 + 5\left(-\frac{26}{13} + \frac{7z}{13}\right) \Rightarrow z = \frac{13 - 13x}{4}$	Obtain 2 correct expressions for one of the variables	A1
	$\frac{x-11}{-\frac{4}{13}} = \frac{y+2}{\frac{7}{13}} = z$	M1 Obtain a Cartesian equation for l A1 Correct equation	M1A1
	$\mathbf{r} = (\mathbf{i} - 2\mathbf{j}) + \lambda\left(-\frac{4}{13}\mathbf{i} + \frac{7}{13}\mathbf{j} + \mathbf{k}\right)$ oe	Deduce a vector equation for l Follow through their Cartesian equation	A1ft
			(5)

(b)	$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$	Correct vector joining P to Q	B1
	$\begin{pmatrix} -4 \\ 7 \\ 13 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -40 \\ 5 \\ -15 \end{pmatrix}$	Attempt vector product between the direction of l and their $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$	M1
		Correct vector	A1
	$\sin \theta = \frac{ -40\mathbf{i} + 5\mathbf{j} - 15\mathbf{k} }{ -4\mathbf{i} + 7\mathbf{j} + 13\mathbf{k} \mathbf{i} + 2\mathbf{j} - 2\mathbf{k} }$	Angle between PQ and line n	
	$d = \overrightarrow{PQ} \sin \theta$		
	$d = \frac{ -40\mathbf{i} + 5\mathbf{j} - 15\mathbf{k} }{ -4\mathbf{i} + 7\mathbf{j} + 13\mathbf{k} } = \frac{1}{\sqrt{234}} \sqrt{40^2 + 5^2 + 15^2}$	Fully correct method for the distance	M1
	$d = \frac{5\sqrt{481}}{39}$	Cao Allow equivalent exact forms e.g. $d = \frac{5\sqrt{74}}{\sqrt{234}}$	A1
			(5)
ALT 1	$\mathbf{r}_m = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -\frac{4}{7} \\ 1 \\ \frac{13}{7} \end{pmatrix} \text{ or } \mathbf{r}_n = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -\frac{4}{7} \\ 1 \\ \frac{13}{7} \end{pmatrix}$	Vector equation for either line with their direction vector from (a)	B1ft
	$\overrightarrow{OP} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \quad \overrightarrow{ON} = \begin{pmatrix} 3 - \frac{4}{7}\mu \\ 2 + \mu \\ 1 + \frac{13}{7}\mu \end{pmatrix} \quad \overrightarrow{NP} = \begin{pmatrix} -1 + \frac{4}{7}\mu \\ -2 - \mu \\ 2 - \frac{13}{7}\mu \end{pmatrix}$	Uses either P and the parametric form of a point on n OR Q and the parametric form of a point on m	
	$\begin{pmatrix} -1 + \frac{4}{7}\mu \\ -2 - \mu \\ 2 - \frac{13}{7}\mu \end{pmatrix} \cdot \begin{pmatrix} -\frac{4}{7} \\ 1 \\ \frac{13}{7} \end{pmatrix} = 0$	M1: Forms scalar product of vector NP and direction vector of l and equates to zero A1: Correct vectors	M1A1
	$\Rightarrow \mu = \frac{56}{117}$	Solves	M1
	$\Rightarrow d = \sqrt{\left(-\frac{85}{117}\right)^2 + \left(-\frac{290}{117}\right)^2 + \left(\frac{10}{9}\right)^2} = \frac{5\sqrt{481}}{39}$	Obtains the correct distance	A1
			(5)
	Alternative for M1A1M1		
	$\overrightarrow{NP} = \begin{pmatrix} -1 + \frac{4}{7}\mu \\ -2 - \mu \\ 2 - \frac{13}{7}\mu \end{pmatrix} \Rightarrow d = \sqrt{\left(-1 + \frac{4}{7}\mu\right)^2 + (-2 - \mu)^2 + \left(2 - \frac{13}{7}\mu\right)^2} \Rightarrow d \text{ is min when } \Rightarrow \mu = \frac{56}{117}$ M1: Find d in terms of a parameter A1: correct expression M1: use calculus (or simplify and complete the square) to find the parameter corresponding to the min d		

ALT 2	Correct vector PQ		B1
	$\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 7 \\ 13 \end{pmatrix} = \begin{vmatrix} 1 \\ 2 \\ -2 \end{vmatrix} \begin{vmatrix} -4 \\ 7 \\ 13 \end{vmatrix} \cos \theta$	Forms the scalar product and attempts to evaluate the LHS	M1
	$\cos \theta = \frac{-16}{3\sqrt{234}}$	Correct value for $\cos \theta$ exact or decimal	A1
	$d = PQ \sin \theta = 3\sqrt{1 - \left(\frac{-16}{3\sqrt{234}}\right)^2} = \frac{5\sqrt{74}}{\sqrt{234}}$	M1: Correct method for the distance. A1: Correct EXACT distance	M1A1
			(5)
			Total 10